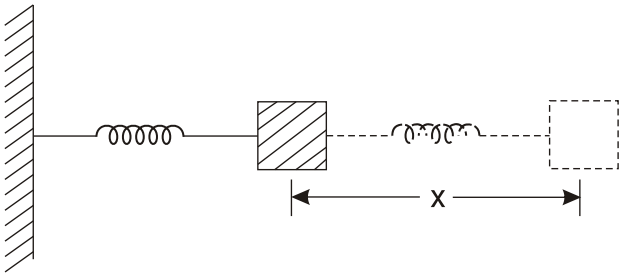
All The Best

Part - A Physics

Section - I: Single Correct

This section contains a total of 20 questions.
All questions in this section are mandatory.
For every correct response you shall be awarded 4 marks.
For every incorrect response -1 marks shall be deducted.
For more details refer the first page of this booklet.

1. The motion of a mass on a spring, with spring constant K is as shown in the figure.



The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$. Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are :

(a) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(b) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(c) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(d) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

2. If in a stationary wave the amplitude corresponding to the antinode is 4 cm, then the amplitude corresponding to a particle of medium located exactly midway between a node and an antinode is

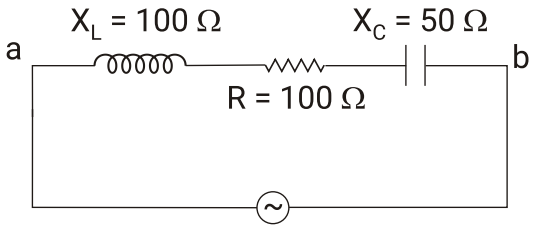
(a) 2 cm

(b) $2\sqrt{2}$ cm

(c) $\sqrt{2}$ cm

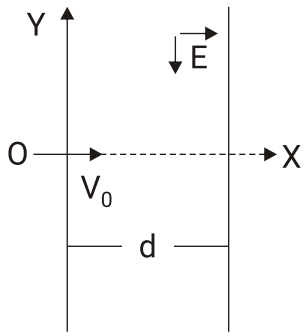
(d) 1.5 cm

3. In the series LCR circuit shown below the peak value of current is $\sqrt{2}$ A. The current at the given instant is 1 A. Find out the possible potential difference $|V_a - V_b|$ at this instant?



(a) 150 V
(b) 250 V
(c) 300 V
(d) Zero

4. A charged particle (mass m and charge q) moves along X-axis with velocity V_0. When it passes through the origin, it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends up to $x = d$. Equation of path of electron in the region $x > d$ is:



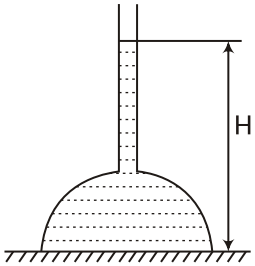
(a) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$

(b) $y = \frac{qEd}{mV_0^2}(x - d)$

(c) $y = \frac{qEd}{mV_0^2}$

(d) $y = \frac{qEd}{mV_0^2}x$

5. A hollow thin hemisphere of mass M and radius R lies on a flat horizontal rubber surface. In the upper hemisphere, there is a small hole in which a long thin tube is inserted (Fig.). The Hemisphere tube is filled with water. To what maximum height can be poured the water into the tube?



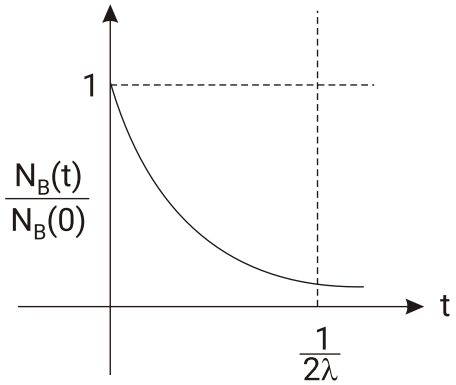
(a) $\frac{\mathrm{M}}{\pi \mathrm{R}^2 \rho}+\frac{2}{3} \mathrm{R}$

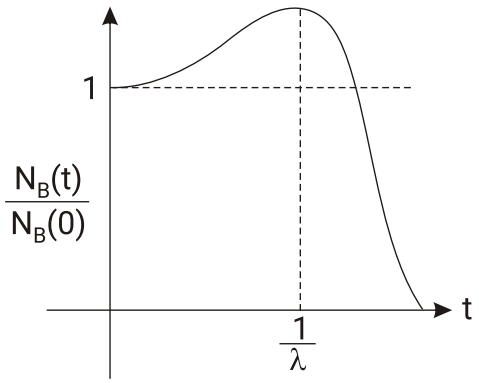
(b) $\frac{\mathrm{M}}{\pi \mathrm{R}^2 \rho}+\frac{1}{3} \mathrm{R}$

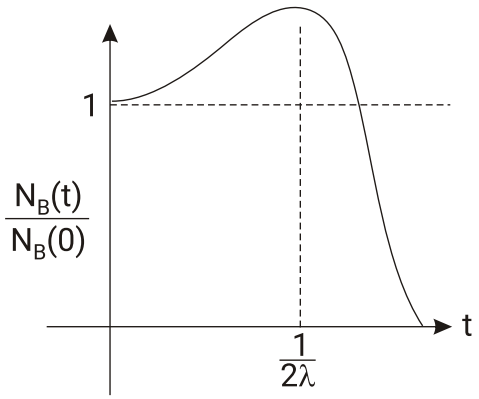
(c) $\frac{\mathrm{M}}{2 \pi \mathrm{R}^2 \rho}+\frac{2}{3} \mathrm{R}$

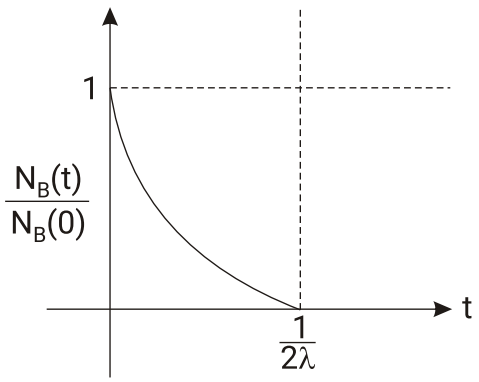
(d) $\frac{\mathrm{M}}{\pi \mathrm{R}^2 \rho}+\frac{4}{3} \mathrm{R}$

6. At time $t = 0$, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $\frac{N_B(t)}{N_B(0)}$ with respect to time t?

(a) 

(b) 

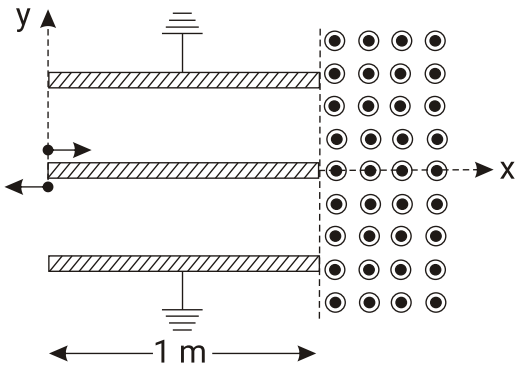
(c) 

(d) 

7. Refractive index of a medium is given by $\mu = \left\{ \begin{matrix} |[y-2]|+1; & 0 < y \leq 5 \\ 4 & y \leq 0, y > 5 \end{matrix} \right\}$, where $[y]$ & $|y|$ represents greatest integer and modulus of y respectively. A light ray coming from the side $y < 0$ in XY plane is incident at $y = 0$ at 30° with Y-axis. Angle made by light ray with Y-axis at $y = 6$ will be

(a) 30°
(b) 45°
(c) 60°
(d) Light cannot come out of $y = 6$.

8. Figure shows three identical plates of plate area of 1 m^2, plate length 1 m and separated by equal separation d. A positively charged particle (of charge q and mass m) with a velocity of $100\hat{i}$ m/s just above the middle plate finally comes out with velocity $100(-\hat{i})$ m/s just below middle plate. For the region $\mathrm{x} > 1$ uniform magnetic field $\overrightarrow{\mathrm{B}} = 1\hat{k}$ T is present. Upper and lower plates are earthed. Gravity, air resistance and edge effect is neglected. What will be the value of charge (in μC upto two decimal places) that should be given to middle plate? (Given that $\frac{\mathrm{q}}{\mathrm{m}} = 1$; assume plate separation is such that charged particle is not hitting the plate)



(a) 18.40
(b) 9.20
(c) 35.40
(d) 36.80

9. If a pushing force making an angle α with the horizontal is applied on a block of mass m placed on a horizontal table and the angle of friction is β, then the minimum magnitude of the force required to move the block is

(a) $\dfrac{\mathrm{mg}\sin\beta}{\cos[\alpha-\beta]}$

(b) $\dfrac{\mathrm{mg}\sin\beta}{\cos[\alpha+\beta]}$

(c) $\dfrac{\mathrm{mg}\sin\beta}{\sin[\alpha-\beta]}$

(d) $\dfrac{\mathrm{mg}\cos\beta}{\cos[\alpha-\beta]}$

10. Consider a spherical gaseous cloud of mass density ρ(r) in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy k. The force acting on the particles is their mutual gravitational force. If ρ(r) is constant in time, the particle number density n(r) = ρ(r)/m is [G is universal gravitational constant]

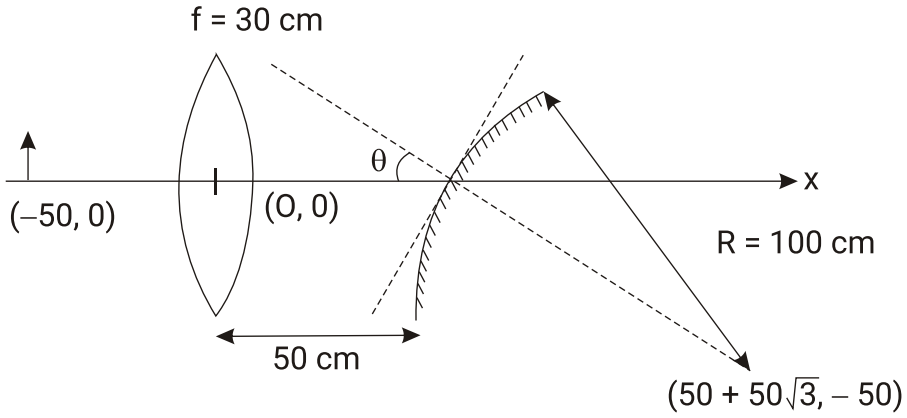
(a) $\frac{3k}{\pi r^2 m^2 G}$

(b) $\frac{k}{2\pi r^2 m^2 G}$

(c) $\frac{k}{6\pi r^2 m^2 G}$

(d) $\frac{k}{\pi r^2 m^2 G}$

11. A smaller object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30^\circ$ to the axis of the lens, as shown in the figure. If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are



(a) $\left(\frac{125}{3}, \frac{25}{\sqrt{3}}\right)$

(b) $\left(25, 25\sqrt{3}\right)$

(c) $\left(50 - 25\sqrt{3}, 25\right)$

(d) $(0, 0)$

12. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube? [Take surface tension of water $T = 7.3 \times 10^{-2}\ Nm^{-1}$, angle of contact = 0, $g = 10\ ms^{-2}$ and density of water = $1.0 \times 10^{3}\ kg\ m^{-3}$]

(a) 3.62 mm

(b) 2.19 mm

(c) 5.34 mm

(d) 4.97 mm

13. A sphere and a cube both made of copper have equal volumes and are blackened. These are heated to the same temperature and are allowed to cool under the same surroundings. The ratio of their initial rates of loss of heat is

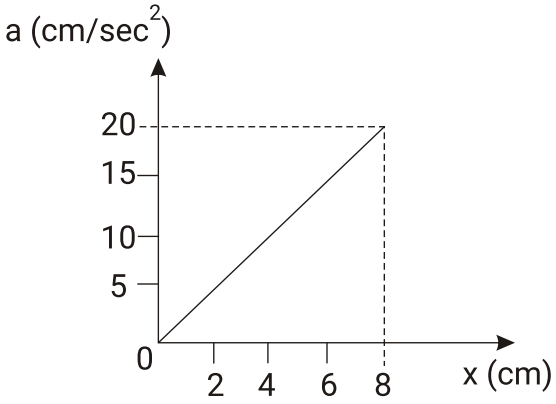
(a) $1:1$

(b) $\left(\frac{\pi}{6}\right)^{\frac{1}{3}}$

(c) $\left(\frac{6}{\pi}\right)^{\frac{1}{3}}$

(d) $\left(\frac{\pi}{6}\right)^{\frac{1}{2}}$

14. A 5 kg mass moves along the x-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass as the mass moves from $x = 0$ to $x = 8$ cm.



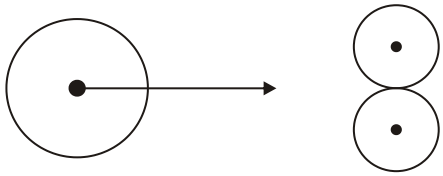
(a) 4×10^{-2} J

(b) 8×10^{-2} J

(c) 2×10^{-2} J

(d) 0.8×10^{-2} J

15. Two equal discs initially at rest are in contact on a table. A third disc of same mass but of double radius strikes them symmetrically and itself comes to rest after impact. Find out the coefficient of restitution?



(a) $\frac{3}{4}$

(b) $\frac{1}{4}$

(c) $\frac{3}{16}$

(d) $\frac{9}{16}$

16. If $y = (\sqrt{ax+b}) \ln x$, find $\frac{dy}{dx}$.

(a) $\frac{\sqrt{ax+b}}{x} + \frac{a \ln x}{2\sqrt{ax+b}}$

(b) $\frac{\sqrt{ax+b}}{x} + \frac{\ln x}{2\sqrt{ax+b}}$

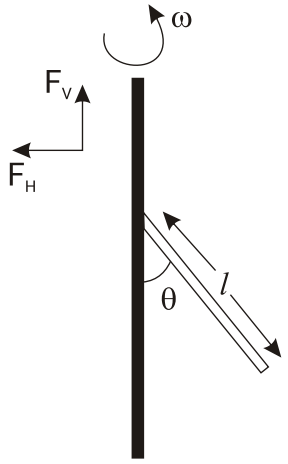
(c) $\frac{\sqrt{ax+b}}{x} - \frac{a \ln x}{2\sqrt{ax+b}}$

(d) None of these

17. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10 °C. Now the end P is maintained at $10^{\circ}C$, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is.

(a) 0.78 mm
(b) 0.90 mm
(c) 1.56 mm
(d) 2.34 mm

18. A uniform rod of length l is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω, the rod makes an angle θ with it (see figure). To find θ, equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (COM) to the torque provided by the horizontal and vertical forces $\mathrm{F_H}$ and $\mathrm{F_v}$ about the COM. The value of θ is then such that:



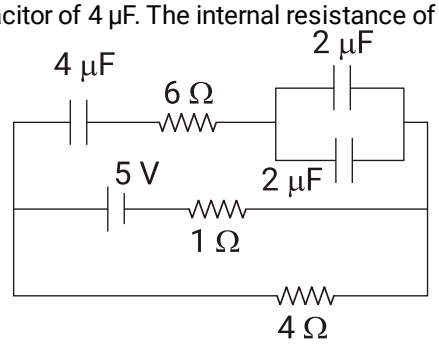
(a) $\cos\theta = \frac{g}{2l\omega^2}$

(b) $\cos\theta = \frac{3g}{2l\omega^2}$

(c) $\cos\theta = \frac{2g}{3l\omega^2}$

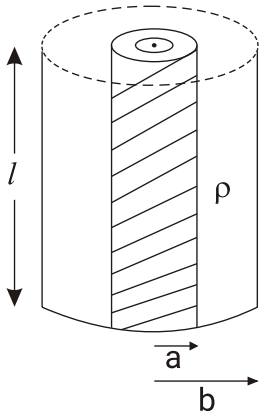
(d) $\cos\theta = \frac{g}{l\omega^2}$

19. Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1Ω:



(a) 8 μC
(b) zero
(c) 16 μC
(d) 4 μC

20. Model a torch battery of length l to be made up of a thin cylindrical bar of radius a and a concentric thin cylindrical shell of radius b filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to the resistance of value R, the maximum Joule heating in R will take place for:-



(a) $\mathrm{R}=\frac{2\rho}{\pi l}\ln\left(\frac{\mathrm{b}}{\mathrm{a}}\right)$

(b) $\mathrm{R}=\frac{\rho}{\pi l}\ln\left(\frac{\mathrm{b}}{\mathrm{a}}\right)$

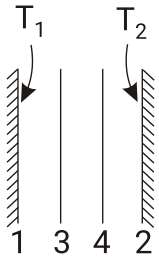
(c) $\mathrm{R}=\frac{\rho}{2\pi l}\left(\frac{\mathrm{b}}{\mathrm{a}}\right)$

(d) $\mathrm{R}=\frac{\rho}{2\pi l}\ln\left(\frac{\mathrm{b}}{\mathrm{a}}\right)$

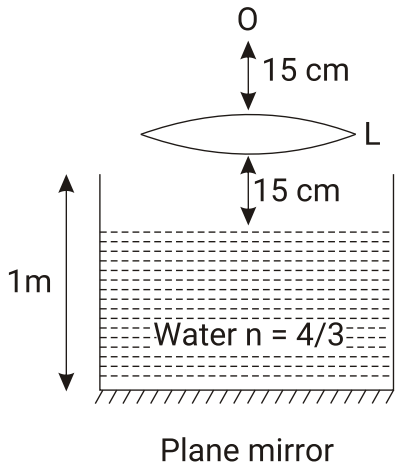
Section - II: Numerical

This section contains a total of 10 questions.
Out of the 10 questions, 5 questions are mandatory.
For every correct response you shall be awarded 4 marks.
For every incorrect response -1 marks shall be deducted.
For more details refer the first page of this booklet.

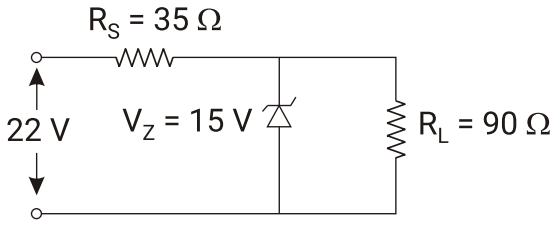
21. As shown in the figure, two large black plane surface are maintained at constant temperature T_1 and T_2 ($T_1 > T_2$). Two thin black plates are placed between the two surfaces and in parallel to them. After some time, stead conditions are obtained. The ratio of heat transfer rate between plate-1 and plate-3 to the ratio of original (when plate-3 and plate-4 was not present) heat transfer rate between plate-1 and plate-2 in steady state is η. Find η?



22. In the figure shown, O is a point object placed above the lens L of focal length 10 cm. Find out the distance between the object O and its image formed in the plane mirror (in cm)?

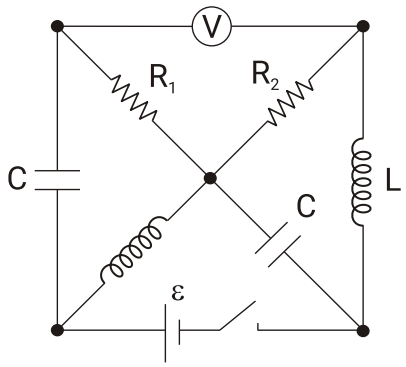


23. The value of power dissipated across the Zener diode (V_Z = 15 V) connected in the circuit as shown in the figure is $x \times 10^{-1}$ watt. The value of x, to the nearest integer, is _____.

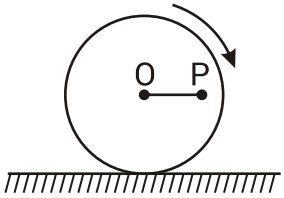


24. In a hydrogen like atom an electron is moving in an orbit having quantum number n. Its frequency of revolution is found to be 13.2×10^{15} Hz. Energy required to move this electron from the atom to the above given orbit is $54.4\ eV$. In a time of 7 nano second the electron jumps back to orbit having quantum number $\frac{n}{2}$. τ be the average torque acted on the electron during the above process, then find $\frac{\tau}{5} \times 10^{27}$ Nm. (given : $\frac{h}{\pi} = 2.1 \times 10^{-34}$ J-s, frequency of revolution of electron in the ground state of H-atom$v_0 = 6.6 \times 10^{15}$ Hz and ionization energy of H-atom, $E_0 = 13.6\ eV$)

25. In the circuit shown below we first put the switch on and wait for the currents to come to a steady-state. Then we put the switch back off. What is the magnitude of the potential difference (in volts) measured by the voltmeter immediately after the switch was turned off? Take the capacitors and coils to be ideal and assume that the voltmeter provides an infinite resistance. Put ε_0 = 1.25 V and $\mathrm{R_1 = 4R_2}$.

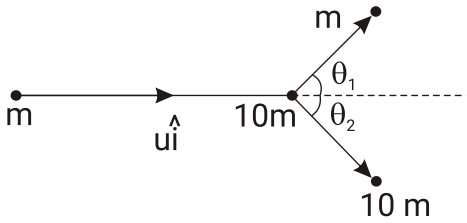


26. A non-uniform disk rolls without slipping on rough ground. The mass of the disk is 5 kg, moment of inertia about O is $\frac{175}{2}$ kgm^2, radius is 3 m. At the instant shown, the centre of mass P lies on a horizontal diameter at a distance of 1 m from O. The angular velocity at this moment is $\frac{1}{\sqrt{5}}$ rad/s. What is the force of friction in N at this moment?

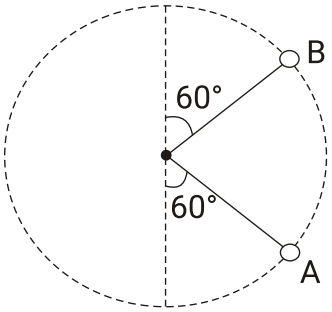


27. A body of mass m is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _____.

28. A particle of mass m is moving along the X-axis with an initial velocity $\mathrm{u}\,\hat{\mathrm{i}}$. It collides elastically with a particle of mass $10\ \mathrm{m}$ at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{\mathrm{n}}\sin\theta_2$, then value of$\mathrm{n}$ is _____.



29. A particle is attached to one end of a string the other end of which is fixed. The particle moves in a vertical plane. The velocity at the lowest point is such that the particle can just complete the circle. If the ratio of the accelerations at A and B is β, then find the value of $38\beta^2$?



30. Seawater at a frequency $f = 9 \times 10^2$ Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is _____ (Given $\frac{1}{4\pi\varepsilon_0}$ = 9 × 10^9 Nm^2C^{-2})

Part - B Chemistry

Section - I: Single Correct

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31. Which of the following reactions does not occur in the Bessemer converter in the extraction of copper from chalcopyrites?

(a) $2CuFeS_2 + O_2 \rightarrow Cu_2S + 2FeS + SO_2$

(b) $FeO + SiO_2 \rightarrow FeSiO_3$

(c) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$

(d) $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

32.

$$\xrightarrow[\text{(excess)}]{+\ CH_2Cl_2\ +\ AlCl_3} X$$

$$X + NBS \longrightarrow Y$$

$$Y \xrightarrow{\text{Hydrolysis}} Z$$

Correct option regarding X, Y, Z.

(a) Z when reacts with lucas reagent, white turbidity is formed within 5-10 minutes.

(b) Maximum number of benzonoid structures X can have is two.

(c) Z when reacts with PCC, benzophenone is formed as product.

(d) Y when reacts with silver nitrate, white precipitate is formed instant.

33. During electrolysis of an aqueous solution of sodium sulphate if 2.4 L of oxygen at STP was liberated at anode. The volume of hydrogen at STP, liberated at cathode would be :

(a) 1.2 L

(b) 2.4 L

(c) 2.6 L

(d) 4.8 L

34. If 3 mole of KI & 2 moles I_2 are reacted with excess of HNO_3 then the Volume of NO_2 gas evolved measured at STP is

Reaction taking place: $KI + I_2 + HNO_3 \longrightarrow HIO_3 + KIO_3 + NO_2 + H_2O$

(a) 716.8 L
(b) 1075.2 L
(c) 44.8 L
(d) 67.2 L

35. $Fe(OH)_2$ is diacidic base has $K_{b1} = 10^{-4}$ and $K_{b2} = 2.5 \times 10^{-8}$
What is the concentration of $Fe(OH)_2$ in 0.1 M $Fe(NO_3)_2$ solution?

(a) 4×10^{-9} M

(b) 2.5×10^{-6} M

(c) 10^{-10} M

(d) 10^{-14} M

36. The density of a gas filled electric lamp is 0.75 kg/m^3. After the lamp has been switched on, the pressure in it increases from 4×10^4 Pa to 9×10^4 Pa. What is increase in U_{rms}

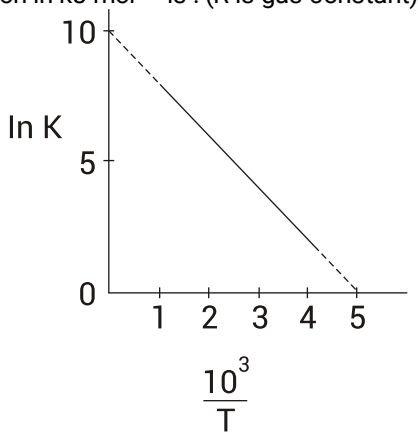
(a) 100 ms^{-1}

(b) 300 ms^{-1}

(c) 200 ms^{-1}

(d) 400 ms^{-1}

37. The rate constant (K) of a reaction is measured at different temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol^{-1} is : (R is gas constant)



(a) 2R
(b) R
(c) 1/R
(d) 2/R

38. $C_6H_6 + A \xrightarrow{AlCl_3} C_6H_5CONH_2$

A in the above reaction is :

(a) NH_2CONH_2

(b) $ClCONH_2$

(c) CH_3CONH_2

(d) $CH_2(Cl)CONH_2$

39. In the manufacturing of Cl_2 gas, according to the reaction: $\overset{*}{H}Cl + O_2 \xrightarrow{CuCl_2} \{\overset{*}{Cl}$ is one isotope of chlorine}

The probable products for this reaction are

(i) $Cl_2 + H_2O$

(ii) $\overset{*}{Cl}_2 + H_2O$

(iii) $\overset{*}{Cl} - Cl + H_2O$

Which of the above three sets of products can actually be obtained?

(a) Only (i)

(b) Both (i) & (ii)

(c) Only (iii)

(d) (i), (ii) & (iii)

40. pH of 0.1 M monobasic acid is measured to be 2. Its osmotic pressure at a given temperature T K is-

(a) 0.1 RT

(b) 0.11 RT

(c) 1.1 RT

(d) 0.01 RT

41. What is the Equivalent weight of H_3PO_4 in the following redox reaction?

$$Cu_3P + Cr_2O_7^{2-} \longrightarrow Cu^{2+} + H_3PO_4 + Cr^{3+}$$

(Assume that Molecular weight of H_3PO_4 is M)

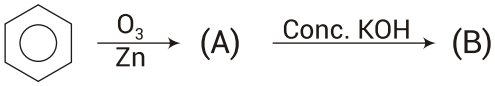
(a) $\frac{M}{3}$

(b) $\frac{M}{6}$

(c) $\frac{M}{8}$

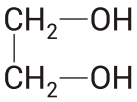
(d) $\frac{M}{11}$

42.

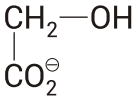


,

Product (B) is:

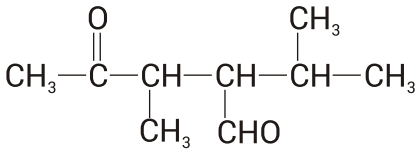
(a) 

(b) 

(c) 

(d) 

43. The IUPAC name of compound



is :

(a) 3,5-Dimethyl-4-formylpentanone

(b) 1-Isopropyl-2-methyl-4-oxobutanal

(c) 2-(1-methyl ethyl)-3-methyl-4-oxopentanal

(d) 3-methyl-2-(1-methylethyl)-4-oxopentanal

44. If x_1, x_2 and x_3 are enthalpies of H—H, O=O and O—H bonds respectively, and x_4 is the enthalpy of vaporisation of water, estimate the standard enthalpy of combustion of hydrogen

(a) $x_1 + \frac{x_2}{2} - 2x_3 + x_4$

(b) $x_1 + \frac{x_2}{2} - 2x_3 - x_4$

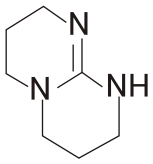
(c) $x_1 + \frac{x_2}{2} - x_3 + x_4$

(d) $2x_3 - x_1 - \frac{x_2}{2} - x_4$

45. The increasing order of pK_b for the following compounds will be :

(A) $\mathrm{NH_2 - CH = NH}$

(B)



(C) $\mathrm{CH_3 - NH - CH_3}$

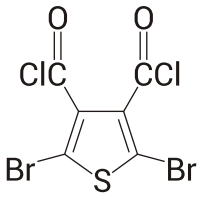
(a) (A) < (B) < (C)

(b) (B) < (C) < (A)

(c) (B) < (A) < (C)

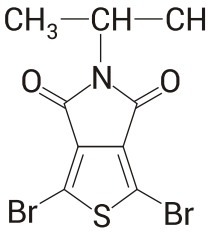
(d) (C) < (A) < (B)

46.

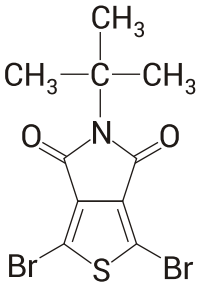


$$+ CH_3CH_2CH_2CH_2NH_2 \xrightarrow{140^oC} X,\ X \text{ is :}$$

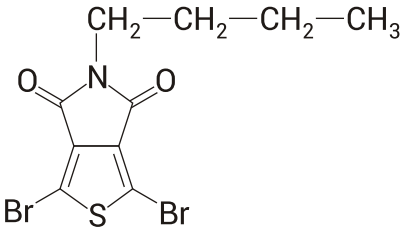
(a)



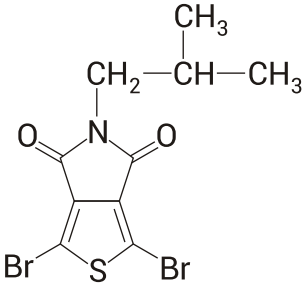
(b)



(c)



(d)



47. Black precipitate of copper sulphide dissolves in:

(a) KCN solution
(b) Sodium sulphide solution
(c) Sodium hydroxide
(d) Boiling dilute (M) sulphuric acid

48. If $\Delta_0 < P$, the correct electronic configuration for d^4 system for an octahedral complex will be –

(a) $t_{2g}^4 e_g^0$

(b) $t_{2g}^3 e_g^1$

(c) $t_{2g}^0 e_g^4$

(d) $t_{2g}^2 e_g^2$

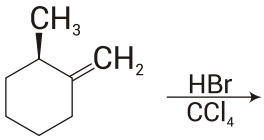
49. A solid AB has rock salt structure. If the edge length is 520 pm and radius of A^+ is 80 pm, the radius of anion B^- would be -

(a) 440 pm

(b) 220 pm

(c) 360 pm

(d) 180 pm

50. 

What is the stereo chemistry of products

(a) Optically inactive

(b) Meso product

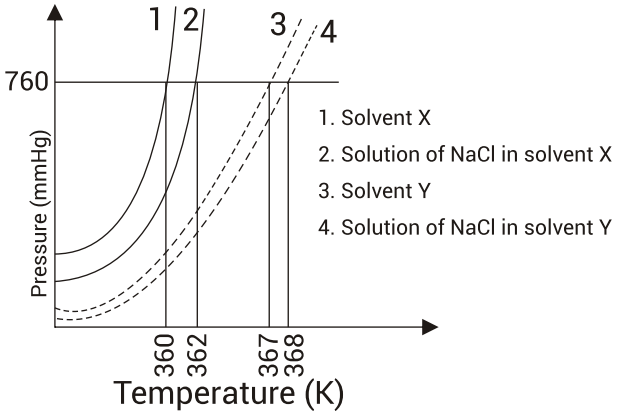
(c) Diastereomers

(d) None of these

Section - II: Numerical

This section contains a total of 10 questions.
Out of the 10 questions, 5 questions are mandatory.
For every correct response you shall be awarded 4 marks.
For every incorrect response -1 marks shall be deducted.
For more details refer the first page of this booklet.

51. The plot given below shows P–T curves (where P is the pressure and T is the temperature) for two solvents X and Y and isomolal solutions of NaCl in these solvents. NaCl completely dissociates in both the solvents.



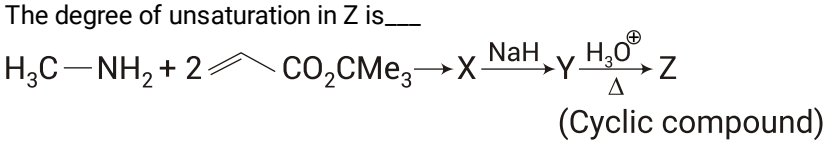
On addition of equal number of moles of a non-volatile solute S in equal amount (in kg) of these solvents, the elevation of boiling point of solvent X is three times that of solvent Y. Solute S is known to undergo dimerization in these solvents. If the degree of dimerization is 0.7 in solvent Y, the degree of dimerization in solvent X is ____.

52. How many of the following salts are white and soluble in dilute HNO_3?
$BaCrO_4$, Hg_2CrO_4, ZnS, $BaSO_4$, BaS_2O_3, CH_3COOAg, $AgNO_2$

53. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ (L = $H_2NCH_2CH_2O^-$) is

54. The molar conductance of 0.05 M solution of $MgCl_2$ is 200 S cm^2mol^{-1} at 25^0C. A cell with electrodes that are 1.50 cm^2 in surface area and 0.50 cm apart is filled with 0.05 M $MgCl_2$ solution. How much current (in A) will flow when the potential difference between the electrodes is 5.0 V?

55. The degree of unsaturation in Z is___



(Cyclic compound)

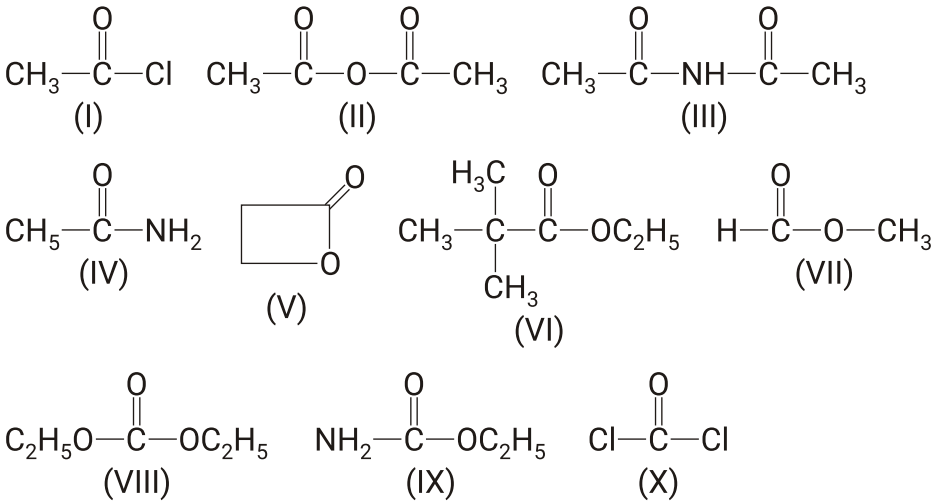
56. An acidified solution of 0.05 M Zn^{2+} is saturated with 0.1 M H_2S. What is the minimum molar concentration (M) of H^+ required to prevent the precipitation of ZnS?
Use $K_{sp}(ZnS) = 1.25 \times 10^{-22}$ and overall dissociation constant of H_2S, $K_{NET} = K_1K_2 = 1 \times 10^{-21}$

57. Consider the kinetic date given in the following table for the reaction A + B + C $\rightarrow$ product.

Exp. No.	[A] ($mol.dm^{-3}$)	[B] ($mol.dm^{-3}$)	[C] ($mol.dm^{-3}$)	[Rate of Reaction] ($mol.dm^{-3}.s^{-1}$)
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for [A] = 0.15 $mol.dm^{-3}$, [B] = 0.25 $mol.dm^{-3}$ and [C] = 0.15 $mol.dm^{-3}$ is found to be $Y \times 10^{-5}$ $mol.dm^{-3}.s^{-1}$.
The value of Y is _____

58. How many compounds out of the following are more reactive than ethyl acetate towards hydrolysis.



59. 20 ml of $Na_2S_2O_3$ solution is required for complete reaction with 40 ml of 0.2 M I_2 solution. What volume (in ml) of 0.5 M $KMnO_4$ solution is needed to react completely with 30 ml of same stock solution of $Na_2S_2O_3$, in weakly basic medium ?

60. Observe the following reactions/statements & find number of CORRECT statements.

(i) SCN^-, CN^- & OCN^- are pseudohalide.

(ii) On oxidation graphite produces $C_6(COOH)_6$

(iii) $I^- \xrightarrow[H_2SO_4]{conc.} I_2$

(iv)

$$Zn \xrightarrow{dil.\ HNO_3} Zn^{2+} + N_2O$$

$$Fe \xrightarrow{dil.\ HNO_3} N_2O + Fe^{3+}$$

$$Ag \xrightarrow{dil.\ HNO_3} NO + Ag^{+}$$

(v) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(vi) $BF_3 < BCl_3 < BBr_3$ (Lewis acidic strength)

(vii) Ionisation energy $O_2 > O$.

(viii) $Xe[PtF_6]$ is the first compound of Xe

(ix) $Si_4O_{11}^{6-}$ is double-chain silicate.

(x) $Mg_2C_3 + 4H_2O \longrightarrow 2Mg(OH)_2 + CH_3\text{-}C{\equiv}C\text{-}H$

Part - C Mathematics

Section - I: Single Correct

This section contains a total of 20 questions.
All questions in this section are mandatory.
For every correct response you shall be awarded 4 marks.
For every incorrect response -1 marks shall be deducted.
For more details refer the first page of this booklet.

61. If $\cos A = \cos B \cos C$ and $A + B + C = \pi$, then the value of $\cot B \cot C$ is

(a) 1

(b) 2

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

62. The value of the integral

$$\int \frac{\sin\theta \cdot \sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^2\theta + 3\sin^2\theta + 6}}{1 - \cos^2\theta} d\theta$$ is :

(where c is a constant of integration)

(a) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(b) $\frac{1}{18}\left[9 - 2\cos^6 - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

(c) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(d) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

63. Let $\theta_1, \theta_2, \ldots\ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots\ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}, z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots\ldots, 10$, where $i = \sqrt{-1}$. Consider the statement P and Q given below:

$P : |z_2 - z_1| + |z_3 - z_2| + \ldots\ldots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots\ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$

Then,

(a) P is TRUE and Q is FALSE

(b) Q is TRUE and P is FALSE

(c) Both P and Q are TRUE

(d) Both P and Q are FALSE

64. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.
Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement from the set G_2 and let S_3 denote the set of these chosen elements.
Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

(a) $\frac{1}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{2}$

(d) $\frac{2}{5}$

65. Let the mirror image of the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ in the plane $ax+by+cz+1=0$ is the line $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-1}{3}$, then the volume of a tetrahedron which the plane makes with the coordinate planes is

(a) $\frac{9}{2}$

(b) $\frac{7}{2}$

(c) $\frac{5}{2}$

(d) None of these

66. The number of solution of the equation $2\cos(e^x) = 5^x + 5^{-x}$, are

(a) No solution
(b) One solution
(c) Two solutions
(d) Infinitely many solutions

67. Two men are walking on a path $x^3 + y^3 = a^3$. When the first man arrives at a point (x_1, y_1), he finds the second man in the direction of his own instantaneous motion. If the coordinates of the second man are (x_2, y_2) and $\frac{x_2}{x_1} \neq \frac{y_2}{y_1}$ then:

(a) $\frac{x_1}{x_2} + \frac{y_1}{y_2} = 0$

(b) $\frac{x_2}{x_1} + \frac{y_2}{y_1} = 0$

(c) $\frac{x_1}{x_2} + \frac{y_1}{y_2} + 1 = 0$

(d) $\frac{x_2}{x_1} + \frac{y_2}{y_1} + 1 = 0$

68. The set of values of x satisfying simultaneously the inequalities $\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5-1)\right)} \geq 0$ and $2^{x-3}-31>0$ is:

(a) a unit set

(b) an empty set

(c) an infinite set

(d) a set consisting of exactly two elements.

69. A tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ meets the ellipse $\frac{x^2}{3a}+\frac{y^2}{2a}=1$ in the points P and Q. If the tangents to the latter ellipse at P and $Q$are at right angles, then a =

(a) 10

(b) 13

(c) 5

(d) $\sqrt{13}$

70. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta, \theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is

(a) $\frac{3}{8}$

(b) $\frac{3}{2}$

(c) $\frac{3}{4}$

(d) $-\frac{3}{4}$

71. If $\lim_{n\to\infty} \dfrac{e\left(1-\dfrac{1}{n}\right)^n - 1}{n^\alpha}$, exists and is equal to $l(l \neq 0)$, then the value of $12(l-\alpha)$ is:

(a) 4

(b) 3

(c) 6

(d) 7

72. If $f:[0,\infty)\to[0,\infty)$ is a function with $f(0)=0$ and $f'(x)>1\,\forall\,x\in(0,\infty)$ then number of solution(s) of the equation $f(x)=f^{-1}(x)$ is

(a) 0
(b) 1
(c) 2
(d) infinite

73. Normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in a point A and let S be its focus. If $|SP|.|SQ|.|SR| = \lambda(SA)^2$, then λ is equal to

(a) a^3

(b) a^2

(c) a

(d) 1

74. The value of $\sum_{n=2}^{\infty} \frac{n}{1+n^2(n^2-2)}$ is equal to

(a) $\frac{5}{4}$

(b) 1

(c) $\frac{5}{16}$

(d) $\frac{1}{4}$

75. The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(a) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

(b) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

(c) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

(d) straight line through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

76. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^x(\log_e \sin x + \cot x)dx =$

(a) $e^{\frac{\pi}{4}} \log_e 2$

(b) $-e^{\frac{\pi}{4}} \log_e 2$

(c) $\frac{1}{2}e^{\frac{\pi}{4}} \log_e 2$

(d) $-\frac{1}{2}e^{\frac{\pi}{4}} \log_e 2$

77. Let $f(x) = (x+1)^2 - 1$, where $x \geq -1$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ is

(a) Empty

(b) $\{0, -1\}$

(c) $\{0, 1, -1\}$

(d) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

78. System of equations $y^{x^2+7x+12} = 1$ and $x + y = 6$, where $x, y \in I$ and $x > 0$, has

(a) no solution
(b) one solution
(c) two solutions
(d) more than two solutions

79. Area bounded by the curve $y^2(2a - x) = x^3$ and the line $x = 2a$ is:

(a) $3\pi a^2$

(b) $\dfrac{3\pi a^2}{2}$

(c) $\dfrac{3\pi a^2}{4}$

(d) None of these

80. In a triangle ABC, if $B = 3C$, then the values of $\sqrt{\left(\frac{b+c}{4c}\right)}$ and $\left(\frac{b-c}{2c}\right)$ are

(a) $\sin C, \sin \frac{A}{2}$

(b) $\cos C, \sin \frac{A}{2}$

(c) $\sin C, \cos \frac{A}{2}$

(d) None of these

Section - II: Numerical

This section contains a total of 10 questions.
Out of the 10 questions, 5 questions are mandatory.
For every correct response you shall be awarded 4 marks.
For every incorrect response -1 marks shall be deducted.
For more details refer the first page of this booklet.

81. Consider, $\alpha = \sin^{-1}\left(\frac{2x}{1+x^2}\right), x \in [-1,1], \beta = \cos^{-1}\left(\frac{3\cos y - 4\sin y}{10}\right), y \in [0, 2\pi]$ and $\gamma = 2\tan^{-1}(z^2 - 4z + 5), z \in R$. If α, β and γ are interior angles of a triangle such that $(\beta + \gamma)$ is minimum then $x + (\tan y) + z = \frac{a - \sqrt{b}}{c}$, where, $a, b, c \in N$. Find the least value of $(a + b + c)$.

82. Find the number of integral values of a for which the numbers 1 and a lie between the numbers x_0 and y_0, where (x_0, y_0) is a solution of the system of equations $x + y - 1 = 2a, 2xy = a^2 - a$.

83. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, where $x, y, z \in N$. If $|adj(adj(adj(adjA)))| = 4^8.5^{16}$, then the number of such matrices A is equal to

($|M|$ represents determinant of a matrix M)

84. Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S :$ sum of entries of A is $7\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals _____

85. Let $f(x)$ and $g(x)$ be two polynomials of degree 2 defined as $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx + r$. Given $a^2 + b^2 + c^2 - 2a + 4b - 2c + 6 = 0$, where $a, b, c \in R$. Also $g(0) = 2, g'(0) = -3$ and $g''(0) = 2$. Find the value of $(f(1) + g(1))$.

86. If p, q are relative prime numbers satisfying $\left(4+\sqrt{15}\right)^{\frac{1}{\log_p(4-\sqrt{15})}+\frac{1}{\log_q\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)}}=\frac{1}{10}$ then find the value of (p^2+q^2)?

87. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m,\ n \in S$ is equal to .....

88. Let $|X|$ denote the number of elements in a set X. Let $S = \{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \leq |B| \leq |A|$ equals

89. The sides a, b and c of triangle ABC satisfy $(a+1)bc = 12, (b+1)ca = 4$ and $(c+1)ab = 4$. If area of triangle equals $\frac{\sqrt{4n^2-1}}{4n}$ where $n \in N$, then find the value of n.

90. let C be the centre of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. The tangents at any point P on this hyperbola meets the straight lines $4x - 3y = 0$ and $4x + 3y = 0$ in the points Q and R respectively. Then $CQ \cdot CR =$